SM3 13.1: Arithmetic Sequences

Let $v = \frac{1}{2}, 9, 32, \ln 17, 4, e^3$. Evaluate the following:

1) Is v arithmetic; if so, find the common difference, d.

No

2) $v_1 = \frac{1}{2}$

3) $v_3 = 32$

4) $v_4 = \ln 17$

Let $a = 2, 7, 12, 17, 22, 27, 32, 37$. Evaluate the following:

5) Is a arithmetic; if so, find the common difference, d.

Yes; $d = 5$

6) $a_1 = 2$

7) $a_3 = 12$

8) $a_5 = 22$

9) $a_9 = \emptyset$

(there are not 9 terms in a)

Let $b = 20, 13, 6, -1, -8, \ldots$. Evaluate the following:

10) Is b arithmetic; if so, find the common difference, d.

Yes; $d = -7$

11) $b_1 = 20$

12) $b_6 = -15$

13)
$$b_{10} = 20 + (10 - 1)(-7)$$
$$b_{10} = 20 + (9)(-7)$$
$$b_{10} = 20 - 63$$
$$b_{10} = -43$$

14)
$$b_{415} = 20 + (415 - 1)(-7)$$
$$b_{415} = 20 + (414)(-7)$$
$$b_{415} = 20 - 2898$$
$$b_{415} = -2878$$

Let $c = 0.3, 1.1, 1.9, \ldots, 80.3, \ldots$. Evaluate the following:

15) Is c arithmetic; if so, find the common difference, d.

Yes; $d = 0.8$

16) $c_1 = 0.3$

17) $c_5 = 3.5$

18)
$$c_n = 80.3$$
$$80.3 = 0.3 + (n - 1)(0.8)$$
$$80.3 = 0.3 + 0.8n - 0.8$$
$$80.3 = -0.5 + 0.8n$$
$$80.8 = 0.8n$$
$$808 = 8n$$
$$101 = n$$

19)
$$c_{29} = 0.3 + (29 - 1)(0.8)$$
$$c_{29} = 0.3 + (28)(0.8)$$
$$c_{29} = 0.3 + 22.4$$
$$c_{29} = 22.7$$

Let $q = \ldots, 2000, 2004, 2008, \ldots$. Evaluate the following:

15) Is q arithmetic; if so, find the common difference, d.

Yes; $d = 4$

16) $q_1 = \emptyset$

The sequence has no starting term.

17) Describe a real world context for what q could represent.

Leap years.

Realize that because the sequence has no start or end, answers that rely on people doing something are likely invalid as people would have to have been doing that activity since the dawn of time and continue doing the activity for the rest of eternity.

William acquires a bag of candy on the evening of Halloween. He eats his candy during the month of November in the following manner: he eats 3 pieces of candy for each day of the month it is (i.e., on the 1st day of November, he eats 3 pieces of candy. On the 2nd day of November, he eats 6 pieces of candy, etc.). Let sequence c represent how many candies are eaten where c_n represents the number of candies eaten on the n^{th} day of November.

18) Is c arithmetic; justify your response.
Yes; c has a common difference

19) How many pieces of candy does William eat on November 17th?
51

20) How many pieces of candy does William eat on November 25th?
75

Mr. Wytiaz decides to lose weight and signs up for a membership at *Linear Weightloss Gym*. After consulting with a personal trainer, Mr. Wytiaz's goal is to lose at least 40 of his 200 pounds. By performing an aerobic workout followed by a kung fu sparring match with Ms. Shaw, he is able to lose $\frac{3}{4}$ of a pound each day. Let sequence w represent the weights of Mr. Wytiaz where w_n represents his weight after his n^{th} session.

21) Is w arithmetic; justify your response.
Yes; w has a common difference

22) $w_1 = 199.25$

23) $w_6 = 195.5$

24) How many sessions until Mr. Wytiaz loses 40 lbs?

A little more than 53 sessions.

The sum of the interior angle measurements of a triangle is 180°, of a convex quadrilateral is 360° and of a convex pentagon is 540°. Let sequence a_n represent the sum of interior angle measurements of a convex polygon with n sides.

25) $a_3 = 180$

26) $a_4 = 360$

27) $a_5 = 540$

28) $a_1 = -180$

29) What does a_1 represent?

nonsense

30) Find the sum of the interior angles of a convex dodecagon ($12gon$).

1800

31) As a_3 represents the polygon with the fewest sides possible, how could we alter our choices of a and n to not include meaningless representations?

We could allow a_3 to become a_1 so that the two meaningless terms are discarded.

32) What is the strongest argument for making the above alteration? What is the strongest argument against making the above alteration?

Pro: No more meaningless terms in our sequence.
Con: n no longer represents both the term index and the number of sides of the polygon.